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Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 4:
Electro-optical distance meters (EDM instruments)

Optique et instruments d'optique — Méthodes d'essai sur site des instruments géodésiques et d'observation —

Partie 4: Télémètres électro-optiques (instruments MED)

Reference number
ISO 17123-4:2001(E)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this part of ISO 17123 may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 17123-4 was prepared by Technical Committee ISO/TC 172, *Optics and optical instruments*, Subcommittee SC 6, *Geodetic and surveying instruments*.

This first edition of ISO 17123-4 cancels and replaces ISO 8322-8:1992 and ISO 12857-3:1997, which have been technically revised.

ISO 17123 consists of the following parts, under the general title *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments*:

— *Part 1: Theory*

— *Part 2: Levels*

— *Part 3: Theodolites*

— *Part 4: Electro-optical distance meters (EDM instruments)*

— *Part 5: Electronic tacheometers*

— *Part 6: Rotating lasers*

— *Part 7: Optical plumbing instruments*

Annexes A and B of this part of ISO 17123 are for information only.

This corrected version of ISO 17123-4:2001 incorporates a correction in the Foreword: ISO 8322-9:1993 has been deleted from the list of documents cancelled and replaced by this part of ISO 17123.

Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 4: Electro-optical distance meters (EDM instruments)

1 Scope

This part of ISO 17123 specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of electro-optical distance meters (EDM instruments) and their ancillary equipment when used in building and surveying measurements. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

This part of ISO 17123 can be thought of as one of the first steps in the process of evaluating the uncertainty of a measurement (more specifically a measurand). The uncertainty of a result of a measurement is dependent on a number of factors. These include among others: repeatability (precision), reproducibility (between day repeatability), traceability (an unbroken chain to national standards) and a thorough assessment of all possible error sources, as prescribed by the ISO Guide to the expression of uncertainty in measurement (GUM).

These field procedures have been developed specifically for *in situ* applications without the need for special ancillary equipment and are purposefully designed to minimize atmospheric influences.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 17123. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 17123 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

ISO 4463-1, *Measurement methods for building — Setting-out and measurement — Part 1: Planning and organization, measuring procedures, acceptance criteria*

ISO 7077, *Measuring methods for building — General principles and procedures for the verification of dimensional compliance*

ISO 7078, *Building construction — Procedures for setting out, measurement and surveying — Vocabulary and guidance notes*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

GUM, *Guide to the expression of uncertainty in measurement*

VIM, *International vocabulary of basic and general terms in metrology*

3 Terms and definitions

For the purposes of this part of ISO 17123, the terms and definitions given in ISO 3534-1, ISO 4463-1, ISO 7077, ISO 7078, ISO 9849, ISO 17123-1, GUM and VIM apply.

4 General

4.1 Requirements

Before commencing surveying, it is important that the operator investigates that the precision in use of the measuring equipment is appropriate to the intended measuring task.

The EDM instrument and its ancillary equipment shall be in known and acceptable states of permanent adjustment according to the methods specified in the manufacturer's handbook, and used with tripods, forced centring equipment and reflectors as recommended by the manufacturer.

The results of these tests are influenced by meteorological conditions. These conditions will include variations in air temperature and air pressure. Actual meteorological data shall be measured in order to derive atmospheric corrections which must be added to the raw distances. The particular conditions to be taken into account may vary depending on where the tasks are to be undertaken. These conditions shall include variations in air temperature, wind speed, cloud cover and visibility. Note should also be taken of the actual weather conditions at the time of measurement and the type of surface above which the measurements are made. The conditions chosen for the tests should match those expected when the intended measuring task is actually carried out (see ISO 7077 and ISO 7078).

Tests performed in laboratories would provide results which are almost unaffected by atmospheric influences, but the costs for such tests are very high, and therefore they are not practicable for most users. In addition, laboratory tests yield precisions much higher than those that can be obtained under field conditions.

This part of ISO 17123 describes two different field procedures as given in clauses 5 and 6. The operator shall choose the procedure which is most relevant to the project's particular requirements.

4.2 Procedure 1: Simplified test procedure

The simplified test procedure provides an estimate as to whether the precision of a given EDM equipment is within the specified permitted deviation according to ISO 4463-1.

The simplified test procedure is based on a limited number of measurements. Therefore, a significant standard deviation cannot be obtained. If a more precise assessment of the EDM instrument under field conditions is required, it is recommended to adopt the more rigorous full test procedure as given in clause 6.

This test procedure relies on having a test field with distances which are accepted as true values. If such a test field is not available, it is necessary to determine the unknown distances, using an EDM instrument of higher precision than that required for the measuring task or using the EDM instrument to be tested immediately after having it calibrated according to the full test procedure as given in clause 6.

4.3 Procedure 2: Full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision of a particular EDM instrument and its ancilliary equipment under field conditions.

The full test procedure is based on measurements of distances in all combinations on a test line without nominal values. The experimental standard deviation of a single distance measurement is determined from a least squares adjustment of the distances in all combinations. Scale errors of an EDM instrument cannot be detected by this procedure. But scale errors do not have any influence either on the experimental standard deviation, s, or on the zero-point correction, δ. In order to determine the stability of the scale, the measuring frequency of the EDM instrument should be checked by means of a frequency meter.

The test procedure given in clause 6 of this part of ISO 17123 is intended for determining the measure of precision in use of a particular EDM instrument. This measure of precision in use is expressed in terms of the experimental standard deviation, s, of a single measured distance:

$s_{\text{ISO-EDM}}$

Further, this procedure may be used to determine:

— the measure of precision in use of EDM instruments by a single survey team with a single instrument and its ancillary equipment at a given time;

— the measure of precision in use of a single instrument over time;

— the measure of precision in use of each of several EDM instruments in order to enable a comparison of their respective achievable precisions to be obtained under similar field conditions.

Statistical tests should be applied to determine whether the experimental standard deviation, s, obtained belongs to the population of the instrumentation's theoretical standard deviation σ, whether two tested samples belong to the same population and whether the zero-point correction, δ, is equal to zero or equal to a predetermined value, δ_0 (see 6.4).

5 Simplified test procedure

5.1 Configuration of the test field

The test field shall consist of one permanently marked instrument station and four permanently mounted reflectors at typical distances for the usual working range of the particular EDM instrument (e.g. from 20 m to 200 m). If permanent mounting of the reflectors is not possible, then the ground points of the reflector stations should be indelibly marked. The reference lengths of the four distances shall be determined as described in 4.2, using an EDM instrument of adequate precision.

In order to set up the test field, each distance shall be measured at least three times and the mean value shall be calculated (see Figure 1). These mean values shall be corrected for deviations in temperature and air pressure from STP (Standard Temperature and Pressure). For this purpose, the air temperature and the air pressure shall be measured in order to determine the necessary corrections of the mean values of the four distances. The mean values shall be corrected by 1 ppm for any deviation of 1 °C in temperature and/or for any deviation of 3 hPa (3 mbar) in air pressure.

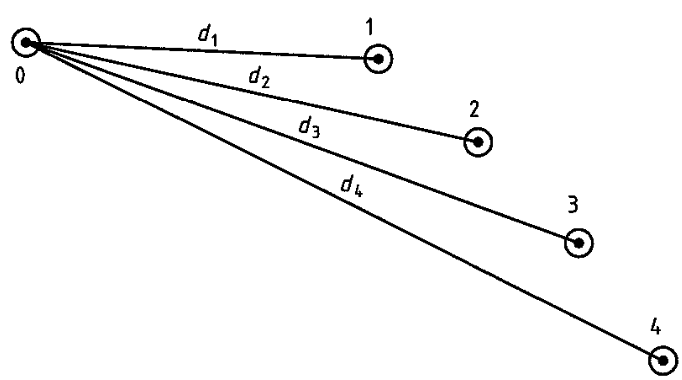

Figure 1 — Configuration of the test field for the simplified test procedure

The corrected mean values of the four distances shall be considered to be true values:

$\overline{x}_1 = d_1$

$\overline{x}_2 = d_2$

$\overline{x}_3 = d_3$

$\overline{x}_4 = d_4$

5.2 Measurements

When setting up the instrument, special care shall be taken when centring above the ground point.

Each distance shall be measured three times. Also, the air temperature and the air pressure shall be measured to derive the atmospheric corrections. The measured distances x_1, x_2, x_3, x_4 are the mean values of the three measurements corrected for atmospheric influences.

5.3 Calculation

All differences $\overline{x}_j - x_j$ shall be within the specified permitted deviation $\pm p$ (according to ISO 4463-1) for the intended measuring task. If p is not given, all differences shall be $|\overline{x}_j - x_j| \leqslant 2{,}5 \times s$, where s is the experimental standard deviation of a single distance measurement, determined according to the full test procedure 2 with the EDM instrument used for determining the distances of the test field.

If the differences $|\overline{x}_j - x_j|$ are too large for the intended task, it is necessary to make further investigations in order to identify the main sources of the errors.

5.4 Further investigations

If all differences $\overline{x}_j - x_j$ have the same sign, then a systematic error is suspected. This can be an error of the zero-point correction or a scale error. If no systematic error can be recognized, then it is recommended to carry out the full test procedure as given in clause 6.

If a scale error is suspected, then the measuring frequency of the EDM instrument should be checked by means of a frequency meter.

To check the zero-point correction, δ, a temporary baseline (about 50 m) consisting of at least three points aligned in the same horizontal plane shall be set out (see Figure 2). Three tripods with forced centring shall constitute the baseline.

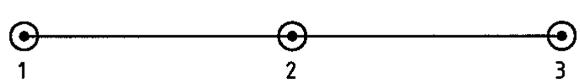

Figure 2 — Temporary base line to check the zero-point correction

From the measured distances between the tripods, the zero-point correction is calculated

$$\delta = \overline{1,3} - \overline{1,2} - \overline{2,3} \qquad (1)$$

where

δ is the zero-point correction;

$\overline{1,3}$, $\overline{1,2}$, $\overline{2,3}$ are the measured distances between the three tripods.

6 Full test procedure

6.1 Configuration of the test line

A straight line approximately 600 m long with seven points shall be established in a horizontal area or in an area with a constant slight slope (see Figure 3). The points shall be stable during the test measurements. In order to obtain representative values for the experimental standard deviation, s, and the zero-point correction, δ, these points shall be selected in such a way that the parts of the measured distances determined by phase measurement with the fine frequency are evenly distributed over the unit length (measuring scale) of the EDM instrument.

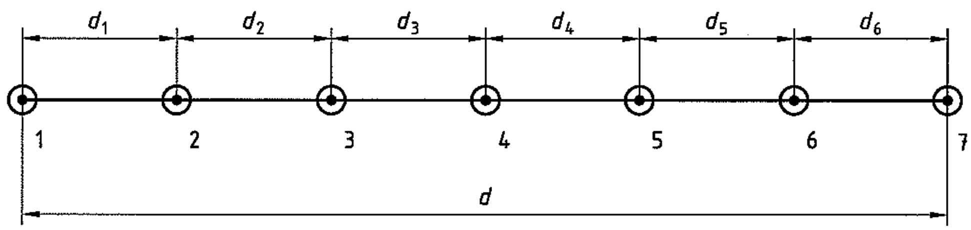

Figure 3 — Configuration of the test line for the full test procedure

A good configuration will be achieved if the six distances $d_1, \ldots, d_6$, between the seven points of the test line are derived by the following procedure:

— $d_0 = 600$ m is the approximate length of the projected test line;

— λ is the wave length of the EDM instrument;

— $\lambda/2$ is the unit length (measuring scale) of the EDM instrument.

$$\beta_0 = \frac{d_0 - 6{,}5\lambda}{15} \quad (2)$$

where β_0 is rounded to the nearest integer value:

$$\beta = \mu \times \lambda/2 \quad (3)$$

where μ is an integer number.

With

$$\gamma = \frac{\lambda}{72} \quad (4)$$

the six distances of the test line and the whole length, d, are calculated:

$$d_1 = \lambda + \beta + 3\gamma \quad (5)$$

$$d_2 = \lambda + 3\beta + 7\gamma$$

$$d_3 = \lambda + 5\beta + 11\gamma$$

$d_4 = \lambda + 4\beta + 9\gamma$

$d_5 = \lambda + 2\beta + 5\gamma$

$d_6 = \lambda + \gamma$

$d = 6\lambda + 15\beta + 36\gamma$

6.2 Measurements

All possible twenty-one distances between the seven points (see Figure 4) shall be measured on the same day. Forced centring interchange should be used to eliminate centring errors. A sufficient number of prisms should ensure that all distances are measured with a good return signal. The measurement of the distances should only be started when the visibility is good and a low insolation is to be expected. The air temperature and pressure should often be measured to ensure that reliable atmospheric corrections can be derived.

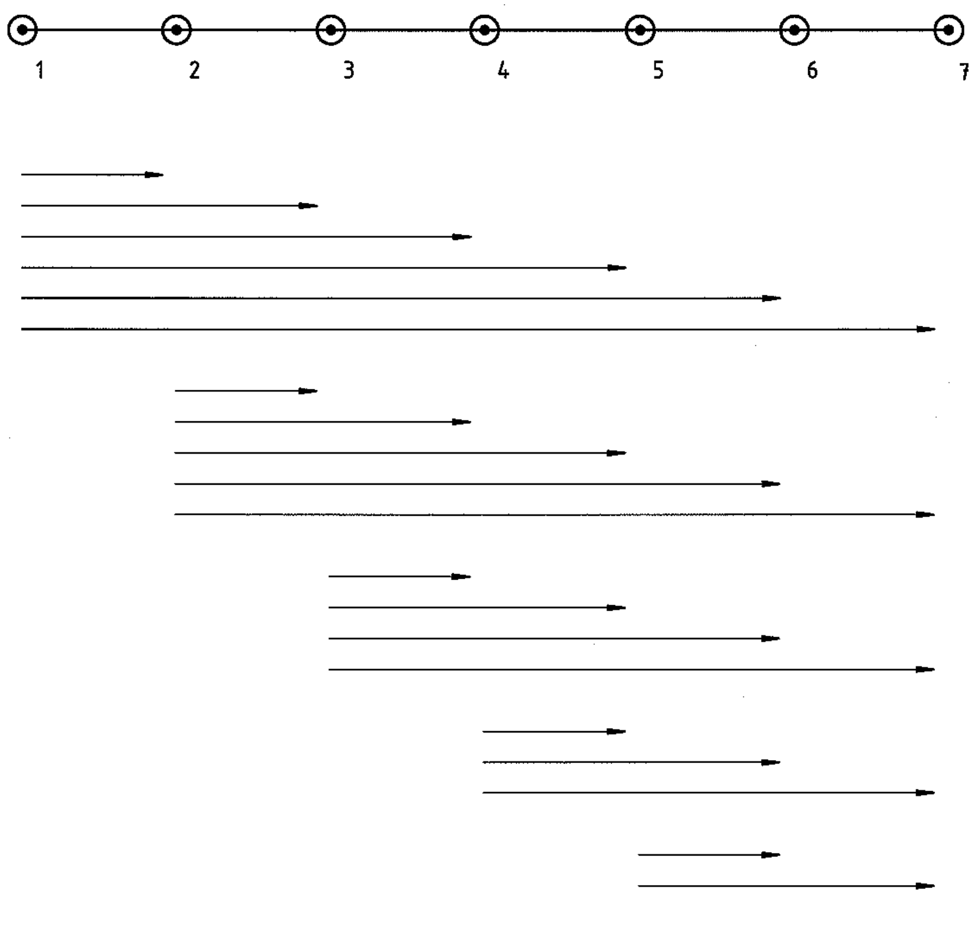

Figure 4 — Distances to be measured

6.3 Calculation

The measurements $\widetilde{x}_{p,q}$ (raw distances = readings on the EDM instrument) shall be corrected for systematic effects (atmospheric correction, slope reduction). These corrected values $x_{p,q}$ are evaluated by an adjustment of observation equations. All weighting factors are set to the unit 1. Unknown parameters are the six distances $\overline{x}_{1,2}, \overline{x}_{2,3}, \ldots, \overline{x}_{6,7}$ and the zero-point correction δ.

The results are derived from:

$$a_p = \sum_{q=1}^{7-p} x_{q,p+q} - \sum_{q=1}^{p} x_{q,7-p+q}; \quad p = 4, 5, 6 \tag{6}$$

$$b_p = \frac{1}{7}\left(\sum_{q=p+1}^{7} x_{p,q} - \sum_{q=1}^{p-1} x_{q,p}\right); \quad p = 1, \ldots, 7 \tag{7}$$

$$\delta = \frac{1}{35}\sum_{p=4}^{6}(2p-7) \times a_p \tag{8}$$

where δ is the zero-point correction.

$$r_{p,q} = b_p - b_q - \frac{7+2(p-q)}{7} \times \delta - x_{p,q}; \quad p = 1, \ldots, 6; \quad q = p+1, \ldots, 7 \tag{9}$$

where $r_{p,q}$ are the residuals of the 21 measured distances $x_{p,q}$ corrected for systematic effects (atmospheric correction, slope correction, but no zero-point reduction).

$$\Sigma r^2 = \sum_{p=1}^{6}\sum_{q=p+1}^{7} r_{p,q} = r_{1,2}^2 + r_{1,3}^2 + \ldots + r_{6,7}^2 \tag{10}$$

where

Σr^2 is the sum of squares of all residuals $r_{p,q}$.

$$\nu = n - u = 14 \tag{11}$$

where

ν is the number of degrees of freedom;

n is the number of measurements ($= 21$);

u is the number of estimated parameters ($= 6 + 1 = 7$).

$$s = \sqrt{\frac{\Sigma r^2}{\nu}} = \sqrt{\frac{\Sigma r^2}{14}} \tag{12}$$

where s is the experimental standard deviation of a single measured distance.

$$s_{\text{ISO-EDM}} = s \tag{13}$$

The experimental standard deviation of the zero-point correction, δ, is derived as follows:

$$s_\delta = s \times \frac{1}{\sqrt{5}} = s \times 0{,}45 \qquad (14)$$

6.4 Statistical tests

6.4.1 General

Statistical tests are recommended for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using

— the experimental standard deviation, s, of a distance measured on the test line, and

— the zero-point correction, δ, of the EDM instrument and its experimental standard deviation, s_δ

in order to answer the following questions (see Table 1):

a) Is the calculated experimental standard deviation, s, smaller than a corresponding value σ stated by the manufacturer or smaller than another predetermined value σ?

b) Do two experimental standard deviations, s and $\widetilde{s}$, as determined from two different samples of measurements belong to the same population, assuming that both samples have the same degree of freedom, ν?

   The experimental standard deviations, s and $\widetilde{s}$, may be obtained from:

   — two samples of measurements by the same instrument at different times;

   — two samples of measurements by different instruments.

c) Is the zero-point correction, δ, equal to zero as supplied by the manufacturer ($\delta_0 = 0$) or, if prisms are used with a given zero-point correction δ_0, is $\delta = \delta_0$?

For the following tests, a confidence level of $1 - \alpha = 0{,}95$ and, according to the design of measurements, a number of degrees of freedom of $\nu = 14$ are assumed.

Table 1 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$s \leqslant \sigma$	$s > \sigma$
b)	$\sigma = \widetilde{\sigma}$	$\sigma \neq \widetilde{\sigma}$
c)	$\delta = \delta_0$	$\delta \neq \delta_0$

6.4.2 Question a)

The null hypothesis stating that the experimental standard deviation, s, is smaller than or equal to a theoretical or a predetermined value σ is not rejected if the following condition is fulfilled:

$$s \leqslant \sigma \times \sqrt{\frac{\chi^2_{1-\alpha}(\nu)}{v}} \qquad (15)$$

$$s \leqslant \sigma \times \sqrt{\frac{\chi^2_{0,95}(14)}{14}} \qquad (16)$$

$$\chi^2_{0,95}(14) = 23{,}68 \qquad (17)$$

$$s \leqslant \sigma \times \sqrt{\frac{23{,}68}{14}} \qquad (18)$$

$$s \leqslant \sigma \times 1{,}30 \tag{19}$$

Otherwise, the null hypothesis is rejected.

6.4.3 Question b)

In the case of two different samples, a test indicates whether the experimental standard deviations, s and $\widetilde{s}$, belong to the same population. The corresponding null hypothesis $\sigma = \widetilde{\sigma}$ is not rejected if the following condition is fulfilled:

$$\frac{1}{F_{1-\alpha/2}(\nu,\nu)} \leqslant \frac{s^2}{\widetilde{s}^2} \leqslant F_{1-\alpha/2}(\nu,\nu) \tag{20}$$

$$\frac{1}{F_{0{,}975}(14{,}14)} \leqslant \frac{s^2}{\widetilde{s}^2} \leqslant F_{0{,}975}(14{,}14) \tag{21}$$

$$F_{0{,}975}(14{,}14) = 2{,}98 \tag{22}$$

$$0{,}34 \leqslant \frac{s^2}{\widetilde{s}^2} \leqslant 2{,}98 \tag{23}$$

Otherwise, the null hypothesis is rejected.

6.4.4 Question c)

The hypothesis of equality of the zero-point corrections δ and δ_0 is not rejected if the following condition is fulfilled:

$$|\delta - \delta_0| \leqslant s_\delta \times t_{1-\alpha/2}(\nu) \tag{24}$$

$$|\delta - \delta_0| \leqslant s_\delta \times t_{0{,}975}(14) \tag{25}$$

$$s_\delta = \frac{s}{\sqrt{5}} = s \times 0{,}45 \tag{26}$$

$$t_{0{,}975}(14) = 2{,}14 \tag{27}$$

$$|\delta - \delta_0| \leqslant \frac{s}{\sqrt{5}} \times 2{,}14 \tag{28}$$

Otherwise, the null hypothesis is rejected.

The number of degrees of freedom and, thus, the corresponding test values $\chi^2_{1-\alpha}(\nu)$, $F_{1-\alpha/2}(\nu,\nu)$, and $t_{1-\alpha/2}(\nu)$ (taken from reference books on statistics) change if a different number of measurements is analysed.

Annex A
(informative)

Example of the simplified test procedure

A.1 Configuration of the test field

An EDM instrument of known precision is used to determine the reference lengths of the four distances of the test field.

The experimental standard deviation of a single measured distance is determined according to the full test procedure as given in clause 6.

$s = 1{,}8$ mm

Reference lengths of the four distances:

$\overline{x}_1 = 21{,}784$ mm

$\overline{x}_2 = 54{,}055$ mm

$\overline{x}_3 = 76{,}502$ mm

$\overline{x}_4 = 152{,}248$ mm

A.2 Measurements

Observer:	S. Miller
Weather:	sunny
	temperature: $+$ 18 °C
	air pressure: 1 009 hPa
Instrument type and number:	NN xxx 630401
Date:	1999-04-15

Table A.1 — Measurements

1 $x_{j,k}$ m	2 $x_j = \frac{\sum_{k=1}^{3} x_{j,k}}{3}$ m	3 $\overline{x}_j$ m	4 $\overline{x}_j - x_j$ mm
21,786	21,785	21,784	−1
785			
785			
54,054	54,053	54,055	2
051			
053			
76,502	76,504	76,502	−2
505			
504			
152,243	152,245	152,248	3
247			
245			

Case 1: The permitted deviation is given: $p = \pm$ 5 mm;

Case 2: p is not given, all differences are $|\overline{x}_j - x_j| <$ 2,5 $\times$ $s =$ 2,5 $\times$ 1,8 mm $=$ 4,5 mm.

In both cases, the EDM instrument is suited for the intended measuring task.

Annex B
(informative)

Example of the full test procedure

B.1 Configuration of the test line

The suggested lengths of the distances $d_1, \ldots, d_6$ are in relation to the total length, d, of the test line and to the length, $\lambda/2$, of the EDM instrument.

According to equations (2) to (4):

$$\beta_0 = \frac{600\ \text{m} - 6{,}5 \times 20\ \text{m}}{15} = 31{,}33\ \text{m}$$

$$\beta = \mu \times 10\ \text{m} = 30{,}00\ \text{m}$$

$$\gamma = \frac{20\ \text{m}}{72} = 0{,}277\,8\ \text{m}$$

where d is approximately 600 m and $\lambda/2$ is 10 m.

With these values, the six distances and the total length, d, of the test line are calculated [according to equation (5)]:

$d_1 =$ 50,84 m
$d_2 =$ 111,96 m
$d_3 =$ 173,08 m
$d_4 =$ 142,52 m
$d_5 =$ 81,40 m
$d_6 =$ 20,28 m
$d =$ 580,08 m

B.2 Measurements

Table B.1 contains, in columns 1 to 4, the twenty-one measured values $x_{p,q}$ corrected for meteorological influences and the slope of the test line.

Observer:	S. Miller
Weather:	sunny
	temperature: +10 °C
	air pressure: 890 hPa
Instrument type and number:	NN xxx 630401
Date:	1999-04-15

Table B.1 — Measurements and residuals

1	2	3	4	5	6	7	8
j	p	q	$x_{p,q}$	$b_p - b_q$	$-\frac{7+2(p-q)}{7} \times \delta$	$r_{p,q}$	$r^2_{p,q}$
			m	m	m	mm	mm^2
1	1	2	50,801	50,808 4	−0,000 9	+2,9	8,41
2	1	3	162,806	162,808 8	−0,000 6	+2,2	4,84
3	1	4	335,904	335,902 7	−0,000 2	−1,5	2,25
4	1	5	478,407	478,401 0	+0,000 2	−5,8	33,64
5	1	6	559,810	559,808 4	+0,000 6	−1,0	1,00
6	1	7	580,098	580,100 1	+0,000 9	+3,0	9,00
7	2	3	112,007	112,004 0	−0,000 9	−3,9	15,21
8	2	4	285,096	285,097 9	−0,000 6	+1,3	1,69
9	2	5	427,594	427,596 2	−0,000 2	+2,0	4,00
10	2	6	509,004	509,003 6	+0,000 2	−0,2	0,04
11	2	7	529,292	529,295 3	+0,000 6	+3,9	15,21
12	3	4	173,091	173,093 9	−0,000 6	+2,0	4,00
13	3	5	315,592	315,592 2	−0,000 9	−0,4	0,16
14	3	6	396,999	396,999 6	−0,000 2	+0,4	0,16
15	3	7	417,295	417,291 3	+0,000 2	−3,5	12,25
16	4	5	142,494	142,498 3	−0,000 9	+3,4	11,56
17	4	6	223,904	223,905 7	−0,000 6	+1,1	1,21
18	4	7	244,200	244,197 4	−0,000 2	−2,8	7,84
19	5	6	81,409	81,407 4	−0,000 9	−2,5	6,25
20	5	7	101,697	101,699 1	−0,000 6	+1,5	2,25
21	6	7	20,292	20,291 7	−0,000 9	−2,2	4,84
Σ						−0,1	145,81

B.3 Calculation

The following values are derived from the corrected measured distances according to equations (6) and (7):

$$a_4 = \sum_{q=1}^{3} x_{q,4+q} - \sum_{q=1}^{4} x_{q,3+q} = 0{,}009\,0 \text{ m}$$

$$a_5 = \sum_{q=1}^{2} x_{q,5+q} - \sum_{q=1}^{5} x_{q,2+q} = 0{,}007\,0 \text{ m}$$

$$a_6 = x_{1,7} - \sum_{q=1}^{6} x_{q,1+q} = 0{,}003\,0 \text{ m}$$

$$b_1 = \frac{1}{7} \sum_{q=2}^{7} x_{1,q} = 309{,}689\,4 \text{ m}$$

$$b_2 = \frac{1}{7}\left(\sum_{q=3}^{7} x_{2,q} - x_{1,2}\right) = 258{,}884\,6 \text{ m}$$

$$b_3 = \frac{1}{7}\left(\sum_{q=4}^{7} x_{3,q} - \sum_{q=1}^{2} x_{q,3}\right) = 146{,}880\,6 \text{ m}$$

$$b_4 = \frac{1}{7}\left(\sum_{q=5}^{7} x_{4,q} - \sum_{q=1}^{3} x_{q,4}\right) = -26{,}213\,3 \text{ m}$$

$$b_5 = \frac{1}{7}\left(\sum_{q=6}^{7} x_{5,q} - \sum_{q=1}^{4} x_{q,5}\right) = -168{,}711\,6 \text{ m}$$

$$b_6 = \frac{1}{7}\left(x_{6,7} - \sum_{q=1}^{5} x_{q,6}\right) = -250{,}119\,0 \text{ m}$$

$$b_7 = -\frac{1}{7}\sum_{q=1}^{6} x_{q,7} = -270{,}410\,7 \text{ m}$$

According to equation (8), the zero-point correction, δ, is calculated:

$$\delta = \frac{1}{35}(a_4 + 3a_5 + 5a_6) = \frac{1}{35} \times 0{,}045 \text{ m} = 1{,}3 \text{ mm}$$

The corrected measured distances, $x_{p,q}$, the zero-point correction, δ, and the values b_p and b_q determine the residuals, $r_{p,q}$, and their square sum according to equations (9) and (10) (see columns 5 to 8 of Table B.1):

$$\Sigma r^2 = 145{,}8 \text{ mm}^2$$

According to the equations (11), (12) and (13), the experimental standard deviation of a single measured distance is calculated:

$$\nu = 14$$

$$s = \sqrt{\frac{145{,}8 \text{ mm}^2}{14}} = 3{,}2 \text{ mm}$$

$$s_{\text{ISO-EDM}} = 3{,}2 \text{ mm}$$

B.4 Statistical tests

B.4.1 Statistical test according to question a)

$$\sigma = 3{,}0 \text{ mm}$$

$$s = 3{,}2 \text{ mm}$$

$$\nu = 14$$

$$3{,}2\text{ mm} \leqslant 3{,}0\text{ mm} \times 1{,}30$$

$$\leqslant 3{,}9\text{ mm}$$

Since the above condition is fulfilled, the null hypothesis stating that the empirically determined experimental standard deviation $s = 3{,}2$ mm is smaller than or equal to the manufacturer's value $\sigma = 3{,}0$ mm is not rejected at the confidence level of 95 %.

B.4.2 Statistical test according to question b)

$$s = 3{,}2\text{ mm}$$

$$\tilde{s} = 4{,}0\text{ mm}$$

$$\nu = 14$$

$$0{,}34 \leqslant \frac{10{,}2\text{ mm}^2}{16{,}0\text{ mm}^2} \leqslant 2{,}98$$

$$0{,}34 \leqslant 0{,}64 \leqslant 2{,}98$$

Since the above condition is fulfilled, the null hypothesis stating that the experimental standard deviations $s_1 = 3{,}2$ mm and $s_2 = 4{,}0$ mm belong to the same population is not rejected at the confidence level of 95 %.

B.4.3 Statistical test according to question c)

$$s = 3{,}2\text{ mm}$$

$$\nu = 14$$

$$\delta = 1{,}3\text{ mm}$$

$$s_\delta = 1{,}4\text{ mm}$$

$$1{,}3\text{ mm} \leqslant 1{,}4\text{ mm} \times 2{,}14$$

$$\leqslant 3{,}0\text{ mm}$$

Since the above condition is fulfilled, the null hypothesis stating that the zero-point correction, δ, is zero is not rejected at the confidence level of 95 %.